4.2 Implicit Differentiation

Question Paper

Course	CIE A Level Maths
Section	4. Differentiation
Topic	4.2 Implicit Differentiation
Difficulty	Hard

Time allowed: 60

Score: /46

Percentage: /100

Question 1

Find an expression for $\frac{dy}{dx}$ in terms of x and y for the following

(i) $2ye^x + 5x^2y^2 = 8$

(ii) $3x \tan y = 2x^2$

[4 marks]

Question 2a

(a) Given that

$$y^2 + 4x^2 - e^y = 0$$

find the positive value of x when $y = 0$

[1 mark]

Question 2b

(b) Hence, or otherwise, find the value of the gradient of

$$y^2 + 4x^2 - e^y = 0$$

at the point where $y = 0$ and x is positive.

[4 marks]

Question 3a

The curve C has equation $2xy^2 - x^2 = 16$
Line L has equation $x = 4$

(a) Show that the two points where C intersects L have equal gradients.

[5 marks]

Question 3b

(b) What else can you deduce about the two points where C and L intercept?

[1 mark]

Question 4

Verify that the point $(-1, 0)$ lies on the curve with equation

$$3xe^y + 2x + 5 = 4y$$

and find the equation of the tangent to the curve at the point $(-1, 0)$.
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[5 marks]

Question 5a

(a) Show that the derivative function of the curve given by

$$\ln y - 2xy^3 = 8$$

is given by

$$\frac{dy}{dx} = \frac{2y^4}{1 - 6xy^3}$$

[5 marks]

Question 5b

(b) Find the equation of the normal to the curve given in part (a) at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

[3 marks]

Question 6

Show that the stationary points on the curve with equation

$$xy^2 - 4x^2 = 64$$

occur when $x = 4$, and find the **exact** y-coordinates of the stationary points.

[6 marks]

Question 7a

(a) Verify that the point $A(1\,,1)$ lies on the curve with equation

$$\ln(xy) + xy^2 = 1$$

[1 mark]

Question 7b

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C. Find the area of the triangle OBC.

[8 marks]

Question 8

Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}[a^{kx}] = ka^{kx}\ln a$$

where a and k are constants.

[3 marks]